Conservation Laws in Economics: What Doesn't Change

Jon Smirl

2026-03-01

The Accountant's Instinct

Every accountant knows the feeling: the balance sheet must balance. Assets equal liabilities plus equity, always. If something appears on one side, something must move on the other. No exceptions.

Conservation laws are the economist's version of this instinct, but applied to the dynamics of how an economy adjusts over time. They tell you what *cannot* happen during adjustment – which quantities are locked in place regardless of how turbulent the economy gets. In a world where everything seems to be in flux, conservation laws identify the things that hold still.

The CES framework produces three distinct conservation laws, each with different economic content and different practical implications. Together they constrain the space of possible adjustments: not everything that looks plausible can actually happen.

Factor Shares Always Sum to One

The simplest conservation law comes from the Euler identity for the CES production function. If F_n is a degree-one homogeneous function of its inputs – meaning that doubling all inputs exactly doubles output – then Euler's theorem guarantees:

$$\sum_{j=1}^{J} \frac{\partial F_n}{\partial x_{nj}} \cdot x_{nj} = F_n$$

Divide both sides by F_n and you get that the factor shares $s_{nj} = (\partial F_n / \partial x_{nj}) \cdot x_{nj} / F_n$ sum to exactly one. Always. Regardless of the curvature parameter ρ, regardless of the number of inputs, regardless of whether the economy is in equilibrium or in crisis.

This is not just a tidy accounting fact. The Euler identity pins the curvature of the *CES potential* $\Phi = -\log F_n$ along the scaling direction (the direction of "make everything proportionally bigger") to the value $1/|\mathbf{x}_n|^2$, completely independent of ρ. The technology parameter, which controls everything about how inputs substitute for one another, has *zero* influence on the curvature along this direction. It is locked by the conservation law.

The practical payoff is immediate. Because this curvature is known without estimating ρ, it provides a model-free way to measure information friction T from aggregate data alone. The coefficient of variation of sector output growth directly reveals T – no production function estimation needed. This is the Euler Pinning result: one conservation law yields one free measurement.

Variance Shifts but Does Not Vanish

The second conservation law concerns the covariance structure of inputs within a sector. At equilibrium under the CES framework, the covariance matrix within a sector takes a highly constrained form called *compound symmetry*:

$$\mathbf{\Sigma}_n = (\sigma_n^2 - \gamma_n)\mathbf{I} + \gamma_n \mathbf{1}\mathbf{1}^\top$$

Every input has the same marginal variance σ_n^2, and every pair of inputs has the same covariance γ_n. This structure has exactly two distinct eigenvalues: one for the "market" direction (all inputs moving together) and $J-1$ identical eigenvalues for the "idiosyncratic" directions (inputs moving relative to each other).

The conservation law here is that the total variance is constrained. Variance can shift between the market direction and the idiosyncratic directions, but the structure pins how it distributes. When information friction rises and the economy approaches the critical boundary, idiosyncratic variance explodes while the market factor stays finite. This is the mechanism behind the well-documented pre-crisis "diversification failure": all assets start moving together not because correlations magically increase, but because the idiosyncratic variance that used to differentiate them has been squeezed out by declining effective curvature.

Example.

Consider an economy with ten manufacturing subsectors. In normal times, each sector has its own idiosyncratic fluctuations – auto production is up while textiles are down, steel is flat while chemicals are booming. The compound symmetry structure says these idiosyncratic movements are approximately equal in magnitude across all pairs.

Now suppose information friction rises (a credit crunch, a breakdown in price signals). The variance does not disappear – it *shifts.* The idiosyncratic differences shrink, and all ten sectors start moving in lockstep. One sector recovering fast while another stays depressed becomes harder: the conservation constraint pushes toward uniform co-movement. The total variance is redistributed, not eliminated.

The Crisis Count Is Fixed

The most surprising conservation law is the *crisis_count_invariant.* Plot the economy's trajectory over time in the space of its two key parameters – the curvature ρ and the effective information friction T_{eff}. A critical curve Γ separates the stable region (below) from the unstable region (above). Each time the trajectory crosses Γ upward and returns, the economy experiences a regime shift – a crisis.

The crisis count invariant $n[\gamma]$ counts how many times the trajectory winds around a point on Γ:

$$n[\gamma] = \frac{1}{2\pi} \oint_\gamma d\theta$$

This number is an integer, and it is *structurally protected*: smooth perturbations to model parameters – fiscal stimulus, monetary policy, regulatory changes – cannot change it. Policy can change *when* a crisis happens and *how severe* it is, but it cannot change *how many* crises occur in a technology cycle unless it fundamentally alters the trajectory's relationship to Γ.

This yields a clean classification. A trajectory with $n = 0$ means a gradual transition with no crisis. $n = 1$ is the standard pattern: one crisis per technology wave (the railroad crash of 1873, the dot-com bust of 2000). $n = 2$ is a double-dip – historically rare but structurally possible when the trajectory lingers near Γ.

The practical implication is sobering. If the economy's technology and institutional parameters put it on an $n = 1$ trajectory, then a crisis will occur somewhere along the way. Policy can delay it (keeping T_{eff} below Γ longer) or soften it (reducing the distance above Γ), but the count is conserved. To change n itself, policy must either prevent the trajectory from crossing Γ at all – effective macroprudential regulation – or move Γ itself by changing the underlying technology or information infrastructure.

Oscillation Energy: The Short-Run Invariant

A fourth approximate conservation law operates on shorter timescales. When sectors are linked by trade (modeled by the antisymmetric coupling matrix $\mathbf{J}$), the oscillation energy

$$\mathcal{L} = \tfrac{1}{2}\boldsymbol{\xi}^\top \mathbf{H}\,\boldsymbol{\xi}$$

is approximately conserved. Here $\boldsymbol{\xi}$ is the displacement from equilibrium and $\mathbf{H}$ is the curvature of the CES potential. The trade linkages conserve $\mathcal{L}$ exactly; only institutional friction $\mathbf{R}$ dissipates it, and slowly.

This explains the familiar pattern of sectoral rotation during business cycles. Manufacturing leads, then services catch up, then finance. One sector's decline is another's gain – not because of some zero-sum competition, but because the oscillation energy is conserved across sectors on short timescales. If one sector recovers unusually fast, the conservation law requires another to recover unusually slowly.

Why Conservation Matters

Conservation laws are constraints on the *possible*. They do not tell you what will happen – they tell you what *cannot* happen. Factor shares cannot sum to anything other than one. Variance cannot appear from nowhere. The number of crises per technology wave cannot be changed by ordinary policy.

These constraints are powerful precisely because they hold regardless of the specific shocks hitting the economy. You do not need to know whether the next crisis will be triggered by a pandemic, a financial bubble, or a trade war. The conservation laws hold in all cases, because they follow from the mathematical structure of CES aggregation itself [@Samuelson1947; @Arrow1961].

For policymakers, the message is both humbling and clarifying. You cannot violate a conservation law. But knowing the constraints tells you where to focus. If you want to reduce the crisis count, work on the trajectory itself – invest in information infrastructure that keeps T_{eff} below the critical curve. If you want to speed recovery in one sector, recognize that the oscillation energy constraint means another sector will bear the cost. And if factor shares must sum to one, then any policy that raises one group's share necessarily reduces another's – no amount of stimulus changes this arithmetic.

Conservation laws are the guardrails of economic dynamics. They cannot be bent, but understanding them makes everything else clearer.

References