

The R_0 Framework: When Growth Becomes Unstoppable

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The Epidemic That Is Not a Disease

In 1927, Kermit McKendrick and William Kermack asked a deceptively simple question: will this outbreak spread, or will it burn itself out? Their answer was a single number, now called R_0 — the basic reproduction number. If each infected person infects more than one other person ($R_0 > 1$), the disease grows exponentially. If each infected person infects fewer than one ($R_0 < 1$), the outbreak shrinks and eventually disappears.

This is not just biology. It is a statement about self-sustaining feedback loops. Any system where one unit of activity generates further activity has its own R_0 . The question is always the same: does each round of the process amplify or diminish?

The CES framework applies this logic to economic transitions. When does a new technology become self-sustaining? When does a distributed network reach critical mass? When does an economic shift become irreversible? The answer, in every case, comes from the same threshold: $R_0 = 1$.

The Next-Generation Matrix

In epidemiology, a disease rarely spreads through a single channel. COVID-19 spread through respiratory droplets, aerosols, surface contact, and asymptomatic transmission. Each channel has its own transmission rate, and the channels interact. To capture this, epidemiologists construct a **next-generation matrix** \mathbf{K} , where entry K_{ij} measures how much activity in channel j generates new activity in channel i (Diekmann1990).

The system's reproduction number is the spectral radius of this matrix — the largest eigenvalue:

$$R_0 = \rho(\mathbf{K})$$

This is not the reproduction number of any single channel. It is the reproduction number of the **system as a whole**, accounting for all cross-channel feedback (VandenDriessche2002).

Definition (Activation Threshold).

A multi-sector system is **self-sustaining** if and only if the spectral radius of its next-generation matrix exceeds one: $\rho(\mathbf{K}) > 1$. Below this threshold, any initial impulse decays. Above it, growth feeds on itself. The *activation_threshold_iff_product* theorem establishes that a nontrivial equilibrium exists if and only if this condition holds.

Why the Whole Exceeds the Parts

Here is the key insight, and the reason this framework matters for economics: **individual sectors can each have $R_0 < 1$, yet the system as a whole can have $R_0 > 1$.**

Consider four channels of technology adoption. Each one, measured in isolation, falls short of self-sustaining:

Channel	Mechanism	Individual R_0
Hardware cost reduction	Learning curve drives prices down	0.7
Network effects	Each new node increases value for all	0.6
Knowledge diffusion	Open-source lowers barriers	0.5
Settlement infrastructure	Payment rails reduce friction	0.4

No single channel sustains itself. Measured alone, each would predict the technology stalls and fades. But the channels do not operate alone. Cheaper hardware attracts more participants (hardware \rightarrow network). More participants produce more open-source contributions (network \rightarrow knowledge). Better knowledge accelerates the learning curve (knowledge \rightarrow hardware). And settlement infrastructure provides the financial plumbing that connects all three to revenue (settlement \rightarrow all).

When we assemble the next-generation matrix with these cross-channel effects, the spectral radius can exceed 1 even though every diagonal entry is below 1. The off-diagonal feedback — the cross-sector amplification — is what pushes the system past the threshold.

This is exactly what the *r0-crossing* describes. The transition does not require any single technology to be independently viable. It requires the **combination** to be viable. The whole exceeds the parts.

The Irreversibility Trap

Once R_0 crosses above 1, something important changes: the transition becomes very hard to reverse.

Suppose a regulator wants to stop the transition by restricting one channel. Export controls on advanced chips, for instance, might reduce the hardware channel's contribution. But if the other three channels compensate — if open-source knowledge accelerates, network effects strengthen, and settlement infrastructure matures — the system's R_0 stays above 1.

To kill a self-sustaining system, you must push R_0 below 1 **simultaneously across all channels**. Reducing one channel while the others adapt is like blocking one door while leaving the windows open.

Example.

Consider COVID-19, with $R_0 \approx 2-3$. Masking reduced aerosol transmission. Social distancing reduced droplet transmission. Surface cleaning reduced contact transmission. Each measure pushed one channel's contribution down. But the virus found alternative routes, and R_0 stayed above 1 in most settings. Only vaccines — which attacked the biological mechanism common to *all* transmission channels — reliably pushed R_0 below 1.

The same logic applies to technology transitions. Export controls on semiconductors to China target one channel. But if knowledge diffusion (open-source models), network

effects (domestic deployment), and alternative hardware (mature-node inference chips) keep the system above threshold, the controls slow the transition without stopping it. The AI mesh finds alternative routes, just as the virus did.

This asymmetry is fundamental. Crossing $R_0 = 1$ from below requires strengthening enough channels simultaneously. Crossing it from above requires weakening *all* channels simultaneously. The threshold is easy to cross in one direction and hard to cross in the other. This is why (Perez2002) documents that technological revolutions, once past their installation phase, have never been reversed — only redirected.

Where the Threshold Sits

The next-generation matrix \mathbf{K} is not a fixed object. Its entries change as technology improves, markets develop, and institutions adapt. The *activation-threshold* framework tracks how R_0 evolves over time.

In the CES hierarchy (see *economy-has-layers*), the four economic levels each contribute to \mathbf{K} . The slowest level — hardware cost, governed by the learning curve — sets the pace. As cumulative production doubles and costs fall by $2^{-\alpha}$ (where $\alpha \approx 0.23$ for semiconductors), the hardware channel’s contribution to \mathbf{K} rises steadily. At some point, the cumulative improvement across all channels pushes $\rho(\mathbf{K})$ past 1.

The *test:tech-cycle-predictions* tracks observable proxies for each channel to estimate where R_0 currently stands. The framework does not predict the exact date of crossing — that depends on investment rates, policy choices, and contingent events. But it does predict the **qualitative character** of the transition: not a gradual adoption curve, but a regime shift from decay to self-sustaining growth, with the asymmetric irreversibility described above.

Cross-Level Amplification

The deepest consequence of the next-generation matrix approach is **cross-level amplification**. In the four-level hierarchy, each level’s equilibrium depends on the level below it (the hierarchical ceiling from *economy-has-layers*). But the R_0 calculation reveals something stronger: feedback between levels can activate the entire system even when every level is individually sub-threshold.

This is not a metaphor. It is a matrix eigenvalue. When the off-diagonal blocks of \mathbf{K} — the terms representing how one level’s activity generates activity at another level — are large enough, the spectral radius exceeds 1 regardless of the diagonal blocks. The system becomes self-sustaining through its connections, not through any component’s individual strength.

For policy, this means that analyzing technologies, sectors, or economic levels in isolation systematically underestimates the potential for transition. The question is never “is this technology viable on its own?” The question is “does this technology, combined with everything it connects to, push R_0 above 1?” (Agrawal2019) frames AI’s economic impact through this lens of complementary capabilities, where the value emerges from combinations rather than components.

The One Number That Matters

R_0 reduces the bewildering complexity of a multi-channel, multi-level economic transition to a single binary question: is the spectral radius above or below 1?

Below 1, the system needs continuous external support — subsidies, mandates, hype cycles — to survive. Remove the support and activity decays. Above 1, the system generates its own momentum. Each round of activity creates more than enough input for the next round. The transition feeds on itself.

The mathematics does not care whether the system is a virus, a technology, or a financial network. The threshold is universal. The only question that matters is which side of it you are on.

References